Hong Kong Mathematics Olympiad (2014 / 2015)

Heat Event (Individual)

香港数学竞赛 (2014 / 2015)

初赛项目(个人)

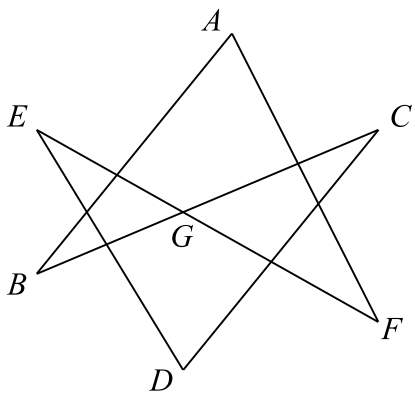
除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

1. 在 1 至 2015 之间（包括 1 及 2015 在内）有多少对相异整数的积是 5 的倍数？
How many pairs of distinct integers between 1 and 2015 inclusively have their products as multiples of 5?

2. 已知 $\left(10^{2015}\right)^{-10^2} = 0.\underbrace{000\cdots0}_{n\text{个}0}1$，求 n 的值。
Given that $\left(10^{2015}\right)^{-10^2} = 0.\underbrace{000\cdots0}_{n\text{ times}}1$. Find the value of n.

3. 设正 n 边形的内角为 $x°$，其中 x 为整数。问 n 有多少个可能值？
Let $x°$ be the measure of an interior angle of an n-sided regular polygon, where x is an integer, how many possible values of n are there?

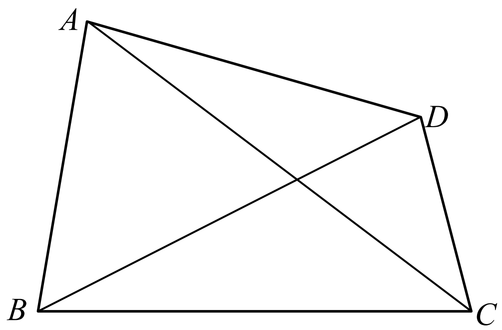
4. 已知下图中 $\angle EGB = 64°$， $\angle A+\angle B+\angle C+\angle D+\angle E+\angle F = ?$
As shown in the figure below that $\angle EGB = 64°$, $\angle A+\angle B+\angle C+\angle D+\angle E+\angle F = ?$



5. 已知 $a_1, a_2, \ldots, a_n, \ldots$ 为一正实数序列，其中 $a_1 = 1$ 及 $a_{n+1} = a_n + \sqrt{a_n} + \frac{1}{4}$。求 a_{2015} 的值。

It is given that $a_1, a_2, \ldots, a_n, \ldots$ is a sequence of positive real numbers such that $a_1 = 1$ and $a_{n+1} = a_n + \sqrt{a_n} + \frac{1}{4}$. Find the value of a_{2015}.

6. 下图中的 $ABCD$ 是一个凸四边形及 $AB+BD+CD=16$，求 $ABCD$ 的最大面积。
As shown in the figure below, $ABCD$ is a convex quadrilateral and $AB+BD+CD=16$. Find the maximum area of $ABCD$.



7. 设 $x, y, z > 1$、$p > 0$、$\log_x p = 18$、$\log_y p = 21$ 及 $\log_{xyz} p = 9$。求 $\log_z p$ 的值。

Let $x, y, z > 1$, $p > 0$, $\log_x p = 18$, $\log_y p = 21$ and $\log_{xyz} p = 9$. Find the value of $\log_z p$.

8. 求 $\frac{1}{4029} + \frac{2\times 2014}{2014^2 + 2015^2} + \frac{4\times 2014^3}{2014^4 + 2015^4} - \frac{8\times 2014^7}{2014^8 - 2015^8}$ 的值。
Find the value of $\frac{1}{4029} + \frac{2\times 2014}{2014^2 + 2015^2} + \frac{4\times 2014^3}{2014^4 + 2015^4} - \frac{8\times 2014^7}{2014^8 - 2015^8}$.

9. 设 x 为实数。求 $\sqrt{x^2-4x+13}+\sqrt{x^2-14x+130}$ 的最小值。

Let x be a real number. Find the minimum value of $\sqrt{x^2-4x+13}+\sqrt{x^2-14x+130}$.

10. 已知 B、H 及 I 为圆上的点。C 是该圆外的一点。BC 是该圆在点 B 的切线。HC 和 IC 分别通过该圆于点 D 及 G。已知 HDC 是 $\angle BCI$ 的角平分线、$\mathrm{BC}=12$、$\mathrm{DC}=6$ 及 $\mathrm{GC}=9$，求 $\dfrac{\Delta BDH\text{ 的面积}}{DHIG\text{ 的面积}}$ 的值。

Given that B, H and I are points on the circle. C is a point outside the circle. BC is tangent to the circle at B. HC and IC cut the circle at D and G respectively. It is given that HDC is the angle bisector of $\angle BCI$, $BC = 12$, $DC = 6$ and $GC = 9$. Find the value of $\dfrac{\text{area of }\Delta BDH}{\text{area of }DHIG}$.

完

END